

## Physics Tool Box

### Solving Newton's Laws of Motion Problems

- Read question to ensure full understanding
- Draw and label a Free Body Diagram
- Separate each FBD for each object as required
- Calculate and Label the x and y components of each force
- Write up the  $\sum F_x = ma_x$  and/or  $\sum F_y = ma_y$  for each group
- Solve above equations separately
- Check validity of your solution

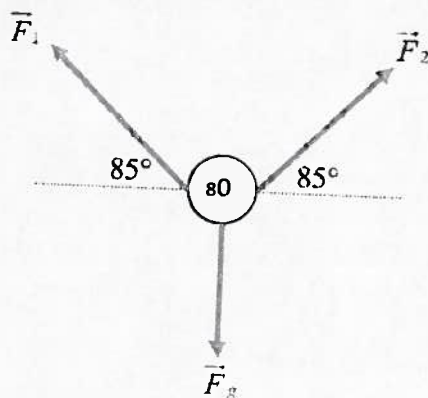
### Types of Questions

- Static questions, objects exerting a force but not moving
  - Objects hanging from ropes
- Kinetic questions
  - Objects being pulled or pushed across floor
  - Gravity moving objects down hill
  - Hanging Weights
  - Inclined Planes

### Example

A gymnast hangs onto two separate rings, attached to the ceiling by ropes. If the person is hanging motionless and each angle is  $85^\circ$ , determine the tension in each rope if the gymnast has a mass of 80kg.

Solution:



We are given:  $m = 80\text{kg}$ ,  $g = 9.8 \frac{\text{m}}{\text{s}^2}$ ,  $\theta = 85^\circ$ ,  $\vec{F}_{\text{net}} = 0$ ,  $F_T = ?$

$$F_{1x} = \cos(85^\circ)$$

$$F_{2x} = \cos(85^\circ)$$

$$F_{1y} = \sin(85^\circ)$$

$$F_{2y} = \sin(85^\circ)$$

Let's sum up force in the x direction

$$F_{\text{net}_x} = F_2 \cos(85^\circ) - F_1 \cos(85^\circ)$$

$$\text{But } 0 = F_2 \cos(85^\circ) - F_1 \cos(85^\circ)$$

$$F_2 \cos(85^\circ) = F_1 \cos(85^\circ)$$

$$F_2 = F_1$$

Let's sum up the force in the y direction

$$F_{\text{net}_y} = F_1 \sin(85^\circ) + F_2 \sin(85^\circ) - (80\text{kg})\left(9.8 \frac{\text{m}}{\text{s}^2}\right) = 2F_T \sin(85^\circ) - 784\text{N}$$

but  $F_{\text{net}_y} = 0$ , therefore

$$2F_T \sin(85^\circ) = 784\text{N}$$

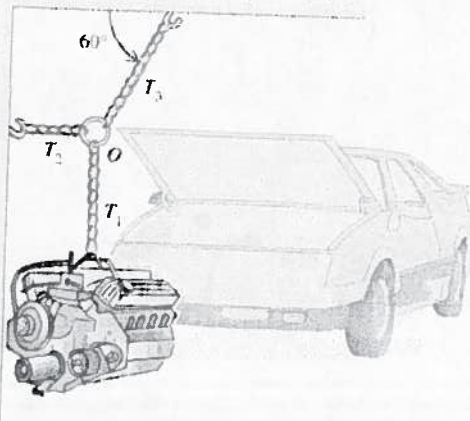
$$F_T = \frac{784\text{N}}{2\sin(85^\circ)} = 393\text{N}$$

2 significant digits

Therefore, the tension in each rope is 390N

### Example

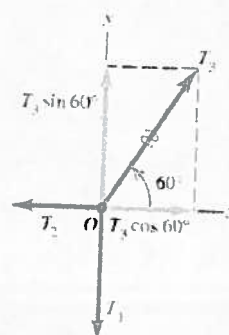
A car engine with weight  $w$  hangs from a chain that is linked at ring O to two other chains, one fastened to the ceiling and the other to the wall. Find the tension in each of the three chains, assuming that  $w$  is the given weight and the rings and chains have no significant weight.



(a)



(b)



(c)

**Solution:**

Note: this question is abstract, we do not know the actual weight of the engine.

Note: because of the ring, the slanted chains do not exert forces on the engine itself, so we will have to separate the forces on the ring from the forces on the engine.

Diagram b) represents the FBD for the engine, while c) the FBD for the ring.

The forces acting on the engine:  $\sum F_y = T_1 + (-w) = 0$  therefore  $T_1 = w$

Let's calculate the forces on the ring

$$\sum F_x = T_3 \cos(60^\circ) + (-T_2) = 0$$

$$\sum F_y = T_3 \sin(60^\circ) + (-T_1) = 0$$

Since  $T_1 = w$ , we can rewrite the y-direction as

$$T_3 \sin(60^\circ) + (-w) = 0$$

$$T_3 = \frac{w}{\sin(60^\circ)} \approx 1.155w$$

$$\begin{aligned} \text{Now for } T_2: T_2 &= T_3 \cos(60^\circ) \\ &= 1.155w \cdot \cos(60^\circ) \\ &\approx 0.577w \end{aligned}$$

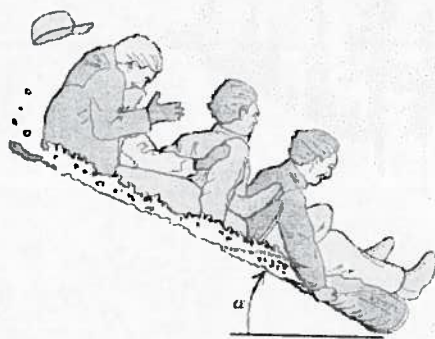
Therefore  $T_1 = w$

$$T_2 = 0.577w$$

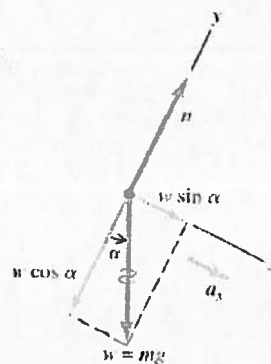
$$T_3 = 1.155w$$

### Example

A toboggan loaded with children slides down a snow covered slope. The combined mass of the children and toboggan is 100kg. The angle  $\alpha$  of the hill is  $30^\circ$ . What is the acceleration of the toboggan?



(a)



(b)

**Solution:**

$$\text{Let the weight } w = mg = (100\text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = 980\text{N}$$

The normal force has only a y-component, but the mass has both an x and y components: Therefore  $w_x = w \sin(30^\circ)$  and  $w_y = -w \cos(30^\circ)$ .

The acceleration is purely in the +x direction, so  $a_y = 0$ . Then from Newton's Second Law we know

$$\sum F_x = w \sin(\alpha) = ma_x$$

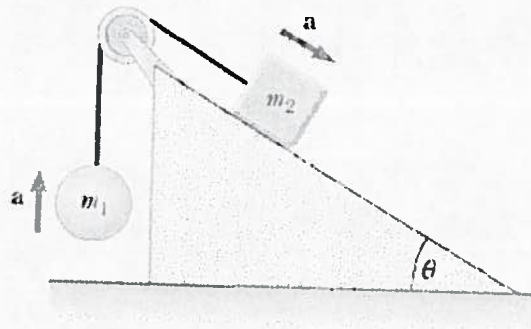
$$\begin{aligned} a_x &= \frac{w \sin(\alpha)}{m} \\ &= g \sin(\alpha) \\ &= 9.8\frac{\text{m}}{\text{s}^2} \sin(30^\circ) \\ &= 4.9\frac{\text{m}}{\text{s}^2} \end{aligned}$$

$$\sum F_y = n - w \cos(\alpha) = ma_y = 0$$

**Note:** we did not need the y component to find the acceleration, which also did not require the mass of the object.

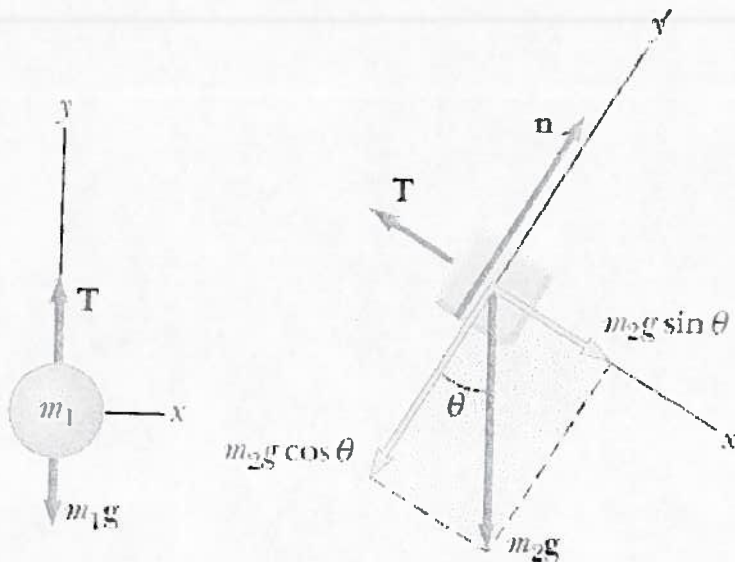
**Example:**

A ball of mass  $m_1=20\text{kg}$  and a block of mass  $m_2=50\text{kg}$  are attached by a lightweight cord that passes over a frictionless pulley of negligible mass. The block lies on a frictionless incline of angle  $30^\circ$ . Find the magnitude of the acceleration of the two objects and the tension in the cord.



**Solution:**

Let's draw free body diagrams for the two masses.



We will attack this problem by recognising that the Tension  $T$  in each free body diagram will be the link that ties the two masses together. So we will let the algebra of Newton's Laws take us to the solution.



Let's look at Newton's Laws when applied to mass 1 where we will decide that acceleration upwards is positive.

$$\sum F_y = m_1 a_y$$

$$-m_1 g + F_{Tension} = m_1 a_y$$

Let's look at Newton's Laws when applied to mass 2 where acceleration down is positive based on the connected movement of the masses (up for hanging mass and down for mass on incline).

$$\sum F_x = m_2 a_x$$

$$-F_{Tension} + m_2 g \sin(\theta) = m_2 a_x$$

Now since the masses are connected, the accelerations are the same. This allows us to solve for the acceleration by adding the two equations (this cancels the tension force).

$$-F_{Tension} + m_2 g \sin(\theta) = m_2 a \quad (1)$$

$$-m_1 g + F_{Tension} = m_1 a \quad (2)$$

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$$m_2 g \sin(\theta) - m_1 g = m_2 a + m_1 a$$

$$m_2 g \sin(\theta) - m_1 g = a(m_1 + m_2)$$

$$a = \frac{m_2 g \sin(\theta) - m_1 g}{(m_1 + m_2)}$$

$$= \frac{(50\text{kg}) \left( 9.8 \frac{\text{N}}{\text{kg}} \right) \sin(30^\circ) - (20\text{kg}) \left( 9.8 \frac{\text{N}}{\text{kg}} \right)}{(50\text{kg} + 20\text{kg})}$$

$$= \frac{49\text{N}}{70\text{kg}}$$

$$= 0.7 \frac{\text{m}}{\text{s}^2}$$

We need only plug in the acceleration into either of the equations and solve for the Tension

$$-m_1 g + F_{Tension} = m_1 a$$

$$F_{Tension} = m_1 a + m_1 g$$

$$F_{Tension} = (20\text{kg}) \left( 0.7 \frac{\text{N}}{\text{kg}} \right) + (20\text{kg}) \left( 9.8 \frac{\text{N}}{\text{kg}} \right)$$

$$= 210\text{N}$$

Therefore the acceleration is  $0.7 \frac{\text{m}}{\text{s}^2}$  and the tension in the rope is 210N.